Synopsis

‘Generalization of Banach Fixed Point Theorem in Complete and Dislocated Quasi-Metric Space’

Submitted to
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स्वामी रामानंद तीर्थ मराठवाडा विद्यापीठ, नांदेड.

For the Degree of
Doctor of Philosophy (Ph.D.)
In Mathematics
(Faculty of Science and Technology)

Submitted by
Mrs. Godavari Ramrao Jojar
(Research Student)
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Under the Guidance of
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December-2020.

Dr. Dolhare U. P.
Professor and Head,
Department of Mathematics
D. S. M. College, Jintur.

To,

The Deputy Registrar (P.G. Section),
S. R. T. M. U.,
Nanded.

Subject: Permission to submit the Synopsis for Doctor of Philosophy in Mathematics.

Respected Sir,

I am recognized guide in Mathematics of our University S. R. T. M. U., Nanded. My candidate **Mrs. Godavari Ramrao Jojar** is submitting registration form for Ph. D. under my guidance. So kindly permit to submit the Synopsis.

All required and needful documents will be fulfill by my candidate.

Thanking you.

Yours Faithfully

Dr. Dolhare U. P.
Professor and Head,
Department of Mathematics

D. S. M. College, Jintur

To,

The Principal,
N. E. S. Science College,
Nanded.

Subject: Permission to submit registration form of Mrs. Godavari Ramrao Jojar.

Respected Sir,

With reference to subject cited above, sir, I am recognized guide in Mathematics of our research center. My candidate Mrs. Godavari Ramrao Jojar is submitting registration form for Ph. D. under my guidance through our research center.

So kindly permit to submit the form.

All required needful documents will be fulfill by my candidate.

Thanking you.

Yours Faithfully

‘Generalization of Banach Fixed Point Theorem in Complete and Dislocated Quasi-Metric Space’

Abstract of Proposed Work Plan:

The fixed point technique is one of the most important and powerful tool that has been generally used for solving the nonlinear operator equation.

The topic of finding the fixed point of the mapping has been the most active area of research work in subject of analysis and topology.

The famous French Mathematician H. Poincare (1854-1912) first recognized the importance of study of nonlinear problems and predicted that the future Mathematics will mainly deal only with the nonlinearity and since then several methods have been developed and employed in the study of nonlinear equations.

As stated by Brown, R. F., Ed “In mathematics, a fixed –point theorem is a result saying that a function F will have at least one fixed point (a point x for which $F(x) = x$), under some conditions on F that can be stated in general terms.” Moreover, a contraction mapping has the one fixed point. Under this, the Banach fixed-point theorem states that each contraction mapping on a nonempty complete topological space which contains a distinctive fixed purpose, that may be for any x in M the iterated perform sequence $x, f(x), f(f(x)), f(f(f(x)))$. Banach fixed-point theorem applies as a proof of the existence of solutions in normal differential equations as well as it is employed as one proof of the function theorem. Therefore, the current work is an attempt of the Generalization of Banach fixed point theorem in dislocated Quasi-metric space and fixed point theorems in complete metric space.

Also I will try to find out relation of generalization of Banach contraction Mapping Principle and fixed point theorems with applications which will be a new generalization in our research work.

Purpose of the Research Topic:

The fixed point theory is an important and major topic of the nonlinear functional analysis and finds some nice applications to other branches of Mathematics for proving the existence and uniqueness theorems to some nonlinear problems that arise in various Physical and Biological processes.

The study of Metric space is one of the best and powerful tool in the use of fixed point theorems. The most of researchers in the subject of Nonlinear Analysis and Topology are studying at the wide range.

In 1906, the Mathematician M. Frechet firstly introduced the notion of metric space. Further development of this notion is developed by the Hausdorff. In 1914, further studies are continued by various authors and develop the metric space and some fixed point theorems under different contraction conditions which are based on the more useful formulation for fixed point theorems.

During the last thirty years, the fixed point theory has been the most active area of research in the subject of Nonlinear Analysis and Topology and at present there is vast literature available in this direction.

The various topological spaces such as Metric space, Complete Metric space, Banach Space, Hilbert space, locally convex and Hausdorff space etc. which have been employed in formulating the fixed point theorems for various classes of contraction mappings.

Recently Rhoades B. E. [9], Dolhare U. P., Dhage B. C., Adrian Petrusel [1] introduced a notion of a new structure of fixed point theorems for the mappings satisfying certain contraction type conditions. Further metric space developed by

Rhoades B. E., Andrian Constantin, Boyd D. W. and Wong J. S. W. [9, 10, 11] , that the celebrated Banach S. [8], Dolhare U. P., Dhage B. C., Lokesh V. and Giniswamy [3], Asad N. A. [6] and Dolhare U. P. [4,5] contracting mapping principle in ordinary metric space follows from the contraction mapping principle in metric space as a special cases. Similarly, it is hoped that the several fixed point theorems in ordinary metric space may follows from the corresponding fixed point theorems in metric space as a special cases. Therefore it is interested to study the fixed point theorems for various classes of mapping in metric as well as Banach Space.

Here, I need to study the non-expansive mappings, Banach fixed point theorem, Strict fixed point theorem, Ciric's fixed points theorem, Khamsi's fixed point theorem, Brouwer fixed point theorem, fixed point theorems for multi valued contractive mappings, contraction mapping principle, Edelstain fixed point theorem, Kranosekii fixed point theorem, Quasi non expansive mappings, Hahn Banach theorem, Boyd and Wong's fixed point theorem, multivalued mappings for fixed point in Complete metric space. Researcher also studies some common fixed points.

Also, we modifying the Banach fixed point theorem in Complete Metric Space and dislocated Metric Space. Also the researcher will study some fixed point theorems for proving fixed point and also we are going to study Banach contraction mapping principle and fixed point theorems with their applications.

In current research study, researcher will also going through the study of fixed point with a well-known classical result about the existence and uniqueness of fixed point of contraction i.e. a function F from Metric space X to itself satisfying the condition

$$d(F(x),F(y)) \le rd(x,y)$$, for all x, y in X, with $0 \le r \le 1$

and try to find out fixed point, generalized fixed point theorems in complete metric space and dislocated Quasi-metric space.

We shall give an application of this result to the solution of fixed point theorems by using the some contractive conditions.

Review of Literature and Development in the Subject (Previous work done in the Relevant area)

Recently, Rhoades B. E. [9], Andrian Petrusel, Dhage B. C. and Dolhare U. P. [1], introduced a notion of a new structure of fixed point theorems for the mapping satisfying certain contraction type conditions. Further Metric space developed by Dhage B. C. and Andrian Constantin [10]. This section is celebrated in D-Metric Space. Asad N. A.[6] generalized fixed point theorems for contracting principle in boundary metric space follows from the contraction mapping principle in metric space as a special case. Similarly, it is hoped that several fixed point theorems in ordinary metric space, Complete Metric Space and dislocated Metric Space are as a special cases. Therefore, we are interested to study the fixed point theorems for various classes for mapping in metric as well as complete space. This part from the following are few studies based on the fixed point theorems and Generalization of Banach contraction mapping principle, and theorems in dislocated Quasi-Metric space.

Objective of Research Study:

Objective of research study are Nine folds.

1) To discuss the fixed points in Metric Space.
2) To discuss the fixed points in non-expansive mapping.
3) To discuss the fixed points for Banach contraction mapping in Metric Space.
4) To discuss the fixed points for contraction in complete metric space.
5) To study of uniform boundedness principle.

6) To study fixed points for bounded metric space.

7) To study the generalization of fixed point theorems in Complete Metric Space and dislocated Quasi-Metric Space.

8) To study the generalization of fixed point theorems in Banach Space and Metris Space.

9) To study the application of fixed point theorems in Metric Space, Complete Metric Space and dislocated Quasi- Metric Space.

Importance of Study/Society Application:

(i) International Status:

Many mathematicians generalized the fixed point theory. Banach S. [8], Khan M. S. and Swaleh M. [13], Cric L. B. [14], Jungck G. and Rhoades B. E. [12], Branciari A. [18] have worked in fixed point theory. In currently decades, Zead Mustafa and Brailey Sims developed terribly nice area i.e. D-Metric space and Generalized many fixed points additionally.

(ii) **National Status:**

Indian Mathematicians additionally operating within the fixed point theory in Metric Space, Complete Metric Space, Dislocated Metric Space etc.. In those studies, Chaudhary B. S., R. K. Bharadwaj, R. Kanan, R. K. Sharma, Bharti Choure, B. Daas, Laxmikanta Devi, C. T. Age, S. Bedre and V. Dapkar, B. C. Dhage developed Metric and Dislocated Quasi-Metric Space which is new literature in Research work.

Significance of Study:

Fixed point theory incorporates wide applications in several fields of Science. There is the most outstanding use of fixed points in Lattice theory. fixed point theory is additionally employed in Political Economy and game theory. The Kakutani fixed point theorems are often accustomed prove the minimal theorem within the theory of complete metric space. Fixed points theory has several applications in arithmetic itself, to find resolution of differential equations to fixed solutions of integral equations etc.

Methodology and Stages in Research Work:

First Stage : In the first stage of my research work, I will try to build up the necessary background to understand the area of my research work and for this purpose, I will read the useful research text books.

Second Stage : Survey of the research papers to collect the literature related to my research problem.

Third Stage : Generalization of new theorems which will be new contribution in research.

Fourth Stage : Fourth stage will be the new results and conclusion.

Fifth Stage : Bibliography and Reference Books.

Plan of Research Work:

1) The first chapter will be introduction, definitions and nonlinear maps.
2) Second chapter will be study of some fixed point and Banach fixed point theorem in Complete Metric Space.

3) Third chapter will be the study of fixed point theorems in dislocated Quasi-Metric Space.
4) Fourth chapter will be the study of fixed point theorem and Mapping principle in Banach Space.
5) Fifth chapter will be new results for fixed point theorems in Complete Metric Space.
6) Sixth chapter will be application of fixed point theorems in Complete Metric Space and dislocated Quasi-Metric Space.
7) References which are used in research work.

Year wise Plan of Work and Target to be Achieved:

First Year: From different research centers, libraries and internet, we will study the basic literature related to the research problem. Researcher will attend and present some papers related to problem in some Journals and conferences.

Second Year: To attend the conference, workshop and seminars in relevant topic and collect research papers for study for further research work.

Third Year: In this stage we will study some fixed point theorems in Metric spaces and its generalizations in Quasi- Metric space and try to generalize them. Also to publish research papers in reputed journals and to find the Application of fixed point theorems in Complete Metric Space and the plan to submit the thesis to University.

References:

[1] **Dolhare U. P., Dhage B. C., Adrian Petrusel,** *Some common fixed point theorems or sequences of Nonself Multivalued operations in Metrically convex D-Metric Spaces*, **Maths, Fixed point Theory, International Journal, Romania, (2003),Volume 4., 143-158.**

[2] **Dolhare U. P., Dhage B. C., and Ntouyas S. K.,** *Existence Theorems for Nonlinear functional Equations in Banach Algebras*, **Communications on Applied nonlinear Analysis, A Great American Journal Volume 10. (2003). Number 4, 59-69.**

[3] **Dolhare U. P., Dhage B. C., Lokesh V. and Giniswamy.,** *Max. Min. principle and contraction mappings in IR* **Maths. Bull. Calcutta, Math. Soc. (2001), Volume 3(2), 332-338.**

[4] **Dolhare U. P.**, *Nonlinear self mapping and some fixed point theorems in D-Metric Spaces*, **Published in Second International Conference of Nonlinear systems, Modeling, Simulation and Application Dec. (2006).**

[5] **Dolhare U. P**., *Generalization of Banach contraction mapping principle*, **Math. Soc. Abd. 1 (2010), 09-12.**

[6] **Assad N. A**., *Fixed valued theorems for set valued transformations on compact set.* **Bull. Un. Math. (4).(1973) L-7.**

[7] **Banach S.,** *Sur Les operation dansles ensembles abstracts et leur application dux integral* **Fund, math.3 (1922), 133-181[9].**

[8] **Banach S.** *Generalized contraction principle*, **Fund Math. 30(1922), 170-184.**

[9] **Rhoades B. E.,** *A comparison of various definition of contraction mapping***Trans, Amer, Math Soc.226 (1977), 257-290.**

[10] **Andrian Constantin (1994)** "*On some fixed point theorems in metric space*" **Univ.U. Novm sadu. Zb. Rad. Prirod-Mat.Fak. Ser. Math. 24, 2 (1994) , 9-21.**

[11] **Boyd D. W. and Wong J. S. W.**, "*A nonlinear constrictions", University of alberta*, **California Institute of Technology, University of Wisconsin and Carnegie-mellon university press,(1969), No. 826.**

[12] **Jungck G. and Rhoades B. E.**, "*Fixed points for set valued functions without continuity*" **Indian J. Pure appl. Math, (March 1998), 29(3):227-238.**

[13] **M. S. Khan and M. Swaleh**: "*Fixed point theorems for generalized contraction*" **Indian J. Pure appl. Math., (September 1984), 15(9):984-990,.**

[14] **Cric L. B. (1973):** "*On a family of contractive maps and fixed points*", **Publication Delinstite Mathematique Nouvello serie tome 17(31), (1974), 45-51.**

[15] **Richard S. Palais**: "*A simple proof of the Banach Contraction principle*", **J. Fixed point theory appl. 2, Birkhauser, Verlag Basel/Switzerland (2007), 221-223(e) .**

[16] **Browder F. E.:** "*Non-expansive non linear operators in Banach Space*", **Proc. Nat Acad. Sci. U.S.A. 54 ,(1965), 1041-1-44.**

[17] **Balkishan Dass, Satya Gupta**, "*An extension of Banach contraction Principle Through Rational Expression*", **Faculty of Mathematics, F. C. Auluck, F.N.A. University of Delhi**.

[18] **A Branciari**, *A fixed point theorem for mappings satisfying a general contractive condition of integral type* **IJMMS(2002), PH Hindawi Publishing Corp, 531-536.**

Signature of Research Scholar **Signature of Research Supervisor**